PHASE VARIATIONS OF OSCILLATIONS IN THE EARTH ORIENTATION PARAMETERS DETECTED BY DIFFERENT TECHNIQUES

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ABSTRACT. The phases of oscillations in the Earth Orientation Parameters (EOP) vary in time. The wavelet transform techniques as well as other methods comprising: complex demodulation (CD), Hilbert transform (HT), Fourier transform band pass and low pass filters (FTBPF and FTLPF) and the least-squares (LS) were applied to compute the phases of the most energetic oscillations in the EOP data. The forced annual oscillations in polar motion and length of day (LOD) are related to the seasonal thermal cycle therefore their phases fluctuate around their well-defined expected values. The expected value of the phase of the Chandler wobble (CW) and of the Free Core Nutation (FCN) is not well-defined, since these oscillations are free wobble oscillations. There is a good agreement between the phases computed by different techniques in the most energetic oscillations of the EOP data.

1. COMPUTATION TECHNIQUES APPLIED

The following computation techniques were applied to determine the instantaneous phases in the EOP time series: 1) Morlet wavelet transform (MWT) (Chui 1992), 2) Harmonic wavelet transform (HWT) (Newland 1998), 3) combination of the FTBPF (Kosek 1995) and CD (FTBPF +CD), 4) combination of the FTBPF and HT (FTBPF+HT), 5) combination of CD and the FTLPF (CD+FTLPF), 6) least-squares (LS) method. All these computation techniques can be applied to complex-valued time series except for the FTBPF+HT technique.

In the MWT and HWT techniques the formula for the transform coefficients, computed as the convolution of the complex-valued signal $x(t)$ and the specific wavelet analyzing function $w(t)$, can be expressed in the frequency domain as follows (Chui 1992, Newland 1998):

$$\widehat{X}(b,T) = \frac{1}{2\pi}|T|^{\frac{1}{2}} \int_{-\infty}^{+\infty} \check{x}(\omega)\overline{\check{w}}(\omega,T)e^{ib\omega}d\omega, \tag{1}$$

where b is the translation (or shift) parameter and $T \neq 0$ is the dilation (or period) parameter, $\check{x}(\omega)$ is the continuous Fourier transform (CFT) of the signal $x(t)$ and $\check{w}(\omega,T)$ is the CFT of the wavelet function applied. In the MWT technique the CFT of the complex-valued Morlet wavelet function (Chui 1992, Schmitz-Hübsch and Schuh 1999)

$$\check{w}(\omega,T) = \sigma[e^{-(\omega T-2\pi)^2\sigma^2/2} - e^{-(\omega T-2\pi)^2\sigma^2/4} \cdot e^{-\pi^2\sigma^2}] \tag{2}$$

is applied, where σ is the parameter, which increase improves the frequency resolution. In the HWT technique the CFT of the harmonic wavelet function

$$\breve{w}(\omega, T) = \begin{cases} \mid T \mid^{-\frac{1}{2}} e^{-(1/T-\omega)^2/(2s^2)} & if \quad |1/T - \omega| \leq \lambda \\ 0 & otherwise \end{cases} \tag{3}$$

localized in the frequency domain near some central frequency $1/T$ is applied, which is of the boxcar type tapered by the gaussian window for better frequency resolution. The frequency resolution is controlled by the window half-width λ and the smoothing parameter s.

Interpretation of the wavelet phase spectrum

$$\widehat{\phi}(b, T) = atan[\Im(\widehat{X}(b, T))/\Re(\widehat{X}(b, T))], \tag{4}$$

where $\Re$ and $\Im$ denote the real and imaginary part of a complex number, respectively, is rather difficult. However, the phase variations can be computed from it for the particular period T_0 by the formula: $\widehat{\varphi}(b, T_0) = \widehat{\phi}(b, T_0) - 2\pi t/T_0$.

To determine the instantaneous phases of an oscillation with central frequency ω_0 in time series $x(t)$ using FTBPF+CD, FTBPF+HT and CD+FTLPF techniques it is necessary to determine first the complex-valued function $z(t, \omega_0)$, from which this phase can be computed as:

$$\widehat{\varphi}(t, \omega_0) = atan[\Im(z(t, \omega_0))/\Re(z(t, \omega_0))]. \tag{5}$$

To compute the function $z(t, \omega_0)$ by the FTBPF+CD technique one must first determine the oscillation with central frequency ω_0 using one- or two-dimensional FTBPF:

$$x(t, \omega_0) = FT^{-1}[FT(x(t)) \cdot A(\omega, \omega_0)], \tag{6}$$

where

$$A(\omega, \omega_0) = \begin{cases} 1 - (\omega - \omega_0)^2/\lambda^2 & if \quad |\omega - \omega_0| \leq \lambda \\ 0 & otherwise \end{cases} \tag{7}$$

is the parabolic transmittance function in which λ is the window half-width which controls the frequency resolution. Next, to remove the linear trend from the instantaneous phases, the oscillation $x(t, \omega_0)$ is multiplied by the monochromatic complex harmonic with the frequency $-\omega_0$: $z(t, \omega_0) = x(t, \omega_0)e^{-i\omega_0 t}$.

In order to compute the function $z(t, \omega_0)$ by the FTBPF+HT technique the oscillation with central frequency ω_0 is first filtered by one-dimensional FTBPF using eqs. (6) and (7). Next, we create the complex-valued series which consists of the oscillation and its HT in the real and imaginary parts, respectively, and from the properties of the HT it follows that it can be also expressed by the formula which involves the Fourier transform (Poularikas 1996):

$$z(t, \omega_0) = x(t, \omega_0) + i \cdot H[x(t, \omega_0)] = FT^{-1}[FT(x(t)) \cdot A(\omega, \omega_0)(sign(\omega) + 1)]. \tag{8}$$

In the CD+FTLPF technique the time series $x(t)$ is first multiplied (demodulated) by the complex-valued harmonic with the frequency $-\omega_0$ (Hasan 1983): $x(t, \omega_0) = x(t) \cdot e^{-i\omega_0 t}$. Next, the transformed signal $x(t, \omega_0)$ is filtered by two-dimensional FTLPF:

$$z(t, \omega_0) = FT^{-1}[FT(x(t, \omega_0)) \cdot A(\omega)], \tag{9}$$

where $A(\omega) = A(\omega, 0)$ is the parabolic transmittance function defined by eq. (7).

2. DATA AND RESULTS

The following EOP series were used in the analysis (IERS 2005): 1) x, y pole coordinates data and LOD data of IERS EOPC04 in 1962.0-2005.6, 2) x, y pole coordinates data of IERS EOPC01 in 1846.0-2002.0, 3) dX, dY IAU2000A nutation-precession corrections in 1979.0 - 2005.6. The x, y pole coordinates data from the EOPC01 and EOPC04 were merged in 1962 to create one data file for the analysis. From the LOD data the IERS Conventions tide model (McCarthy and Petit 2004) has been removed to get the LODR data.

Next, the phase variations of the most energetic oscillations in the EOP were computed by different techniques and to enable their comparison their mean values were subtracted. Figure 1 shows the phase variations of the Chandler and annual oscillations computed in x pole coordinate data by the MWT, CD+FTLPF and FTBPF+CD techniques. Figure 2 shows the phase variations of the semi-annual oscillations computed in complex-valued $x - iy$ pole coordinate data by the MWT, HWT and FTBPF+CD techniques. The boundary effects of the HWT technique cause irregular jumps of the computed phase at the ends of data time span. Figure 3 shows the phase variations of the annual oscillation computed in LODR data by the MWT, CD+FTLPF and FTBPF+HT techniques. Figure 4 shows the phase variations of the FCN computed in complex-valued $dX + idY$ IAU2000A nutation-precession corrections by the LS, MWT and FTBPF+CD techniques.

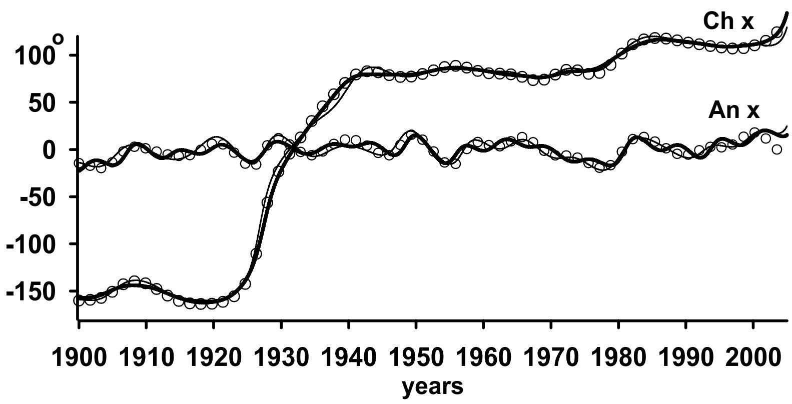

Fig. 1. Phase variations of the Chandler and annual oscillations computed in x pole coordinate data by the MWT ($\sigma = 2$) (thick line), CD+FTLPF ($\lambda = 0.0004$) (circles) and FTBPF+CD ($\lambda = 0.0004$)(thin line) techniques.

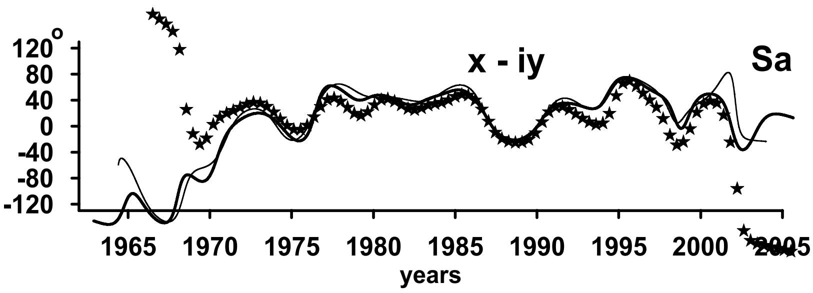

Fig. 2. Phase variations of the semi-annual oscillation computed in complex-valued $x - iy$ pole coordinate data by the MWT ($\sigma = 1$)(thick line), HWT ($\lambda = 0.002, s = 0.003$) (stars) and FTBPF+CD ($\lambda = 0.001$)(thin line) techniques.

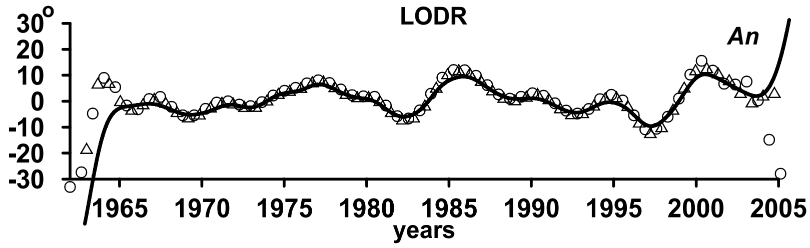

Fig. 3. Phase variations of the annual oscillation computed in LODR data by the MWT ($\sigma = 1$)(thick line), CD+FTLPF ($\lambda = 0.001$) (cirles) and FTBPF+HT ($\lambda = 0.001$) (triangles) techniques.

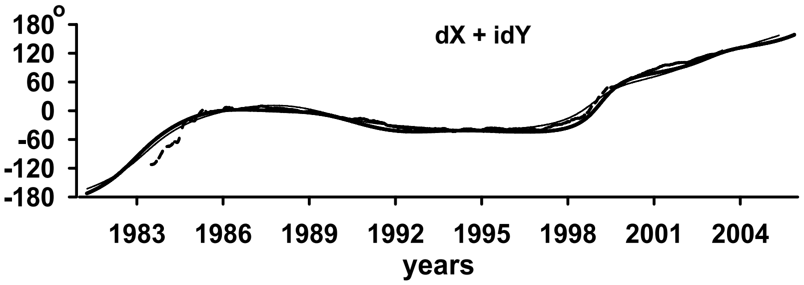

Fig. 4. Phase variations of the FCN computed in complex-valued $dX+idY$ IAU2000A nutation-precession corrections by the LS in 5 year time intervals (dashed line), MWT ($\sigma = 1$)(thick line) and FTBPF+CD ($\lambda = 0.0005$) (thin line) techniques.

3. DISSCUSSION

The phase variations of the most energetic oscillations in the EOP are very important for studying the Chandler wobble excitation (Kosek 2005) or their influence on the EOP prediction errors (Kosek et al. 2002). The phases of the most energetic oscillations in the EOP computed by the presented techniques are in a good agreement, except at the ends of data time span where boundary effects occur, especially for the HWT technique.

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